A Gallina generating backend to check OCaml's type inference correctness

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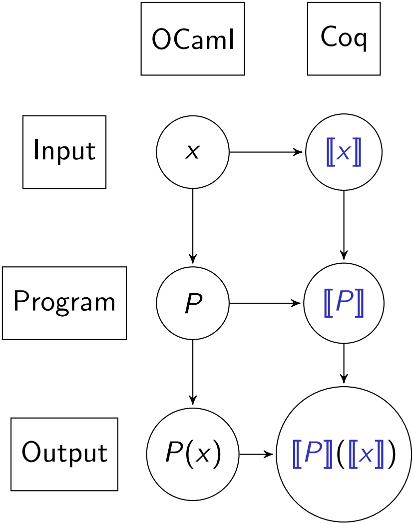
Starting point

- Proving the correctness of the full OCaml type inference is hard
- We can prove it theoretically for subparts, but combining them is complex
- Writing a type checker for the typed syntax tree might help, but still suffers the same diffculties
- Alternative approach: ensure that the generated typed syntax trees enjoys type soundness by translating them into another type system

Soundness by translation



If for all $P : \tau \to \tau'$ and $x : \tau$

- P translates to $[\![P]\!]$, and $\vdash [\![P]\!] : [\![\tau \to \tau']\!]$
- x translates to $[\![x]\!]$, and $\vdash [\![x]\!] : [\![\tau]\!]$
- $[\![P]\!]$ applied to $[\![x]\!]$ evaluates to $[\![P(x)]\!]$
- $[\![\cdot]\!]$ is injective (on types)

then the soundness of Coq's type system implies the soundness of OCaml's evaluation

Requirements for soundness

- Need to evaluate programs, so no axioms in translated programs
- Need to preserve Coq's soundness, so avoid other axioms too
- Must implement OCaml's features, such as references, or polymorphic comparison inside Coq
- In turn this requires an intensional representation of OCaml's types, to be able to use them in computations

Overview

- Define a type representing OCaml types: `ml_type`
- And a translation function `coq_type : ml_type -> Type`
  This function must be computable.
- Wrap mutability and failure/non-termination into a monad

```
Definition M T := Env -> Env * (T + Exn).
```

- `Env` is a mapping from keys (which contain some `T : ml_type`) to values of type `coq_type T`.
  The definition of `Env` needs to bypass the positivity check.
- As a result one can write non-terminating programs in Coq, but we think that since env contains only ML values, this does not make Coq incoherent.
- No other axiom or bypassing is used (at this point).

Definition of `ml_type`

`ml_type` is just an inductive type with a branch for each OCaml type constructor used in the program. For instance:

```
Inductive ml_type :=
  | ml_int                (* predefined types *)
  | ml_exn
  | ml_arrow (_ : ml_type) (_ : ml_type)
  | ml_ref (_ : ml_type)
  | ml_list (_ : ml_type)
  | ...
  | ml_color              (* types from the program *)
  | ml_tree (_ : ml_type) (_ : ml_type)
  | ml_ref_vals (_ : ml_type).
```

Since it is used as a parameter for all polymorphic definitions, it needs to be defined first, but depends on nothing else.
Decidable equality is generated automatically by tactics.

Translation of type definitions

- ML types have two representions in Coq: an intensional one as a term `t : ml_type`, and a shallow embedding `coq_type t`.
- In order to infer type equalities, some embedded types need to refer to intensional representations:

```
loc    : ml_type -> Type      (* translation of 'a ref *)
newref : forall (T : ml_type), coq_type T -> M (loc T)
```

- This creates a problem when translating polymorphic type definitions, as their type variables may be used either in an intensional or extensional way, and `coq_type` is not yet defined.
- Solution: use separate type parameters for intensional and extensional occurrences.

```
(* type 'a ref_vals = RefVal of 'a ref * 'a list *)
Inductive ref_vals (a : Type) (a_1 : ml_type) :=
  RefVal (_ : loc a_1) (_ : list a).
```

Definition of coq_type

Once we have translated the type definitions, coq_types can be generated:

```
Variable M : Type -> Type.      (* The monad is not yet defined *)
Fixpoint coq_type (T : ml_type) : Type :=
  match T with
  | ml_int => Int63.int
  | ml_exn => ml_exns
  | ml_arrow T1 T2 => coq_type T1 -> M (coq_type T2)
  | ml_ref T1 => loc T1
  | ml_list T1 => list (coq_type T1)
  | ...
  | ml_color => color
  | ml_tree T1 T2 => tree (coq_type T1) (coq_type T2)
  | ml_ref_vals T1 => ref_vals (coq_type T1) T1
```

Thanks to this definition, polymorphic values need only take the intensional representation as parameter.

Building the execution monad

We can now build the monad, by applying a predefined functor, which takes ml_type and coq_type as parameters.

```
Record binding (M : Type -> Type) := mkbind
  { bind_key : key; bind_val : coq_type M (key_type bind_key) }.
Inductive Exn := Catchable of ml_exns | GasExhausted | ...
Definition M0 Env T := Env -> Env * (T + Exn).
#[bypass_check(positivity)]          (* non-positive definition *)
Inductive Env := mkEnv : int -> seq (binding (M0 Env)) -> Env.

Definition M T := M0 Env T.
Definition Ret {A} (x : A) : M A := fun env => (env, inl x).
Definition Fail {A} (e : Exn) : M A := fun env => (env, inr e).
Definition Bind {A B} (x : M A) (f : A -> M B) : M B :=
  fun env => match x env with
             | (env', inl a) => f a env'
             | (env', inr e) => (env', inr e)
             end.
```

Purity analysis

- For each definition, we compute its *pure arity*, i.e. the number of applications before it may exhibit impure behavior.
- We use it to avoid turning all arrows into monadic ones.
- To avoid purity polymorphism, all function arguments are assumed to be values of pure arity 1.

```
type ('a,'b) tree =
  Leaf of 'a | Node of ('a,'b) tree * 'b * ('a,'b) tree ;;

let mknode t1 t2 = Node (t1, 0, t2) ;;          (* pure arity = 3 *)

Inductive tree (a : Type) (b : Type) :=
  | Leaf (_ : a)
  | Node (_ : tree a b) (_ : b) (_ : tree a b).

Definition mknode (T : ml_type) (t1 t2 : coq_type (ml_tree T ml_int))
  : coq_type (ml_tree T ml_int) :=
  Node (coq_type T) (coq_type ml_int) t1 0%int63 t2.
```

Translating recursive functions

To allow the translation of arbitrary recursive functions, all recursive functions take a gas parameter, and as a result may raise the exception GasExhausted.

```
let rec mccarthy_m n =                    (* pure arity = 1 *)
  if n > 100 then n - 10
  else mccarthy_m (mccarthy_m (n + 11));;
```

```
Fixpoint mccarthy_m (h : nat) (n : coq_type ml_int)
  : M (coq_type ml_int) :=
  if h is h.+1 then
    do v <- ml_gt h ml_int n 100%int63;     (* comparison *)
    if v then Ret (Int63.sub n 10%int63) else
      do v <- mccarthy_m h (Int63.add n 11%int63);
      mccarthy_m h v
  else Fail GasExhausted.
```

Comparison functions

OCaml allows polymorphic comparison. We mimic it by generating a type analyzing function.

```
Fixpoint compare_rec (h : nat) (T : ml_type)
  : coq_type T -> coq_type T -> M comparison :=
  if h is h.+1 then
    match T as T return coq_type T -> coq_type T -> M comparison with
    | ml_int => fun x y => Ret (Int63.compare x y)
    | ml_arrow T1 T2 =>                         (* fail as in OCaml *)
      fun x y => Fail (Catchable (Invalid_argument "compare"%string))
    | ml_ref T1 =>                  (* compare contents of references *)
      fun x y => compare_ref (compare_rec h) T1 x y
    | ml_ref_vals T1 => fun x y =>
        match x, y with RefVal x1 x2, RefVal y1 y2 =>
          lexi_compare (compare_rec h (ml_ref T1) x1 y1)
            (Delay (compare_rec h (ml_list T1) x2 y2))
        end
    ...
    end
  else fun _ _ => FailGas.
```

Breaking strong normalization...

The seemingly innocuous non-positive definition of `Env` allows to define really non-termination functions (without gas).

```
let omega x =
  let r = ref (fun x -> x) in
  let delta y = !r y in
  r := delta; delta x ;;
```

```
Definition omega (T : ml_type) (x : coq_type T) : M (coq_type T) :=
  do r <- newref (ml_arrow T T)
                 (fun x : coq_type T => Ret (x : coq_type T));
  let delta (y : coq_type T) : M (coq_type T) :=
    AppM (getref (ml_arrow T T) r) y in
  do _ <- setref (ml_arrow T T) r delta; delta x.
```

Note that one still needs to use a reference, so this can only be done inside the monad. That is why we believe that one cannot use this to prove `False`.

Simulating the toplevel

Contrary to C, OCaml allows toplevel statements (of pure arity 0) to change the global state. This is tricky to do this in Coq.

```
let r = ref [3] ;;
let z = r := 1 :: !r; !r;;

Definition Restart {A B} (x : W A) (f : M B) : W B :=
  BindW (fun _ => x) (fun _ => f).                  (* W for Writer monad *)
Definition it : W unit := (empty_env, inl tt).

Definition r :=
  Restart it (newref (ml_list ml_int) (3%int63 :: @nil (coq_type ml_int))).
Definition z :=
  Restart r           (* the same state should only be restarted once! *)
    (do r <- FromW r;               (* can access the value repeatedly *)
     do _ <- (do v <- (do v <- getref (ml_list ml_int) r;
                        Ret (@cons (coq_type ml_int) 1%int63 v));
              setref (ml_list ml_int) r v);
     getref (ml_list ml_int) r).
Eval vm_compute in z.
```

How to use

- New backend to OCaml, defined in the `ocaml_in_coq` branch of `COCTI/ocaml` on GitHub. (PR #3)

  `https://github.com/COCTI/ocaml/pull/3`
- Adds a `-coq` option to `ocamlc`, which switches to the Coq generation backend, producing a `.v` rather than a `.cmo`.
- At this point, supports only single file programs written in core ML plus references and algebraic datatypes (sum types), using a subset of `Pervasives`

Related work

- Guillaume Claret. *Coq of OCaml*. OCaml Workshop, 2014.
- Antal Spector-Zabusky *et al. Total Haskell is reasonable Coq*. CPP, 2018.
- Danil Annenkov *et al. ConCert: a smart contract certification framework in Coq*. CPP, 2020.
- Laila El-Beheiry *et al.* SMLtoCoq: Automated Generation of Coq Specifications and Proof Obligations from SML Programs with Contracts. LFMTP, 2021.
- Matthieu Sozeau *et al. Coq Coq correct! verification of type checking and erasure for Coq, in Coq*, POPL, 2020.
- Pierrick Couderc. *Vérification des résultats de l'inférence de types du langage OCaml.* PhD Thesis, Université Paris-Saclay, 2018.

Prospects

- Could also be used to do proofs about the translated programs, using the Monae library [Affeldt et al., 2019]
- We first plan to add our monad to the Monae hierarchy
- The use of an intentional representation for ML types should allow to properly translate GADTs
- Anybody interested ?

Demo